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Full-System Linearization for Wind Turbines with Aeroelastically Tailored Rotor Blades in OpenFAST

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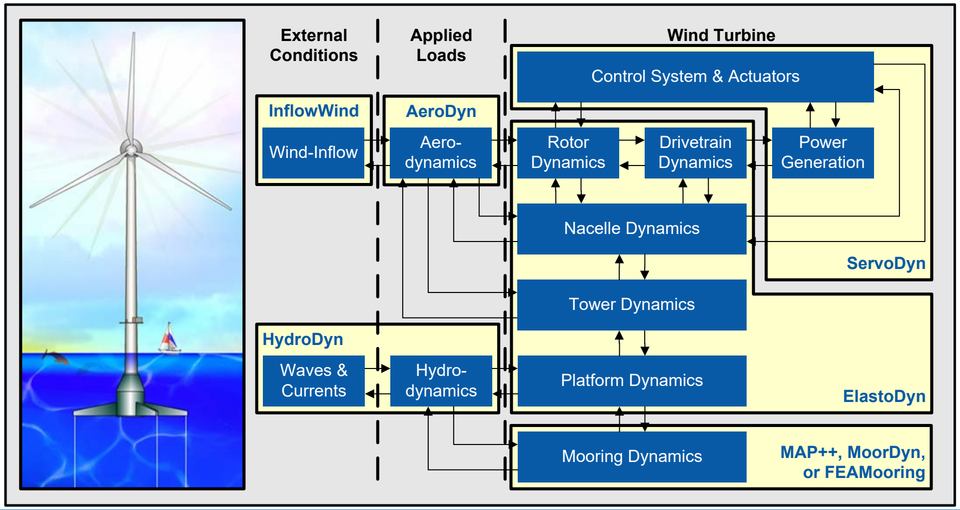
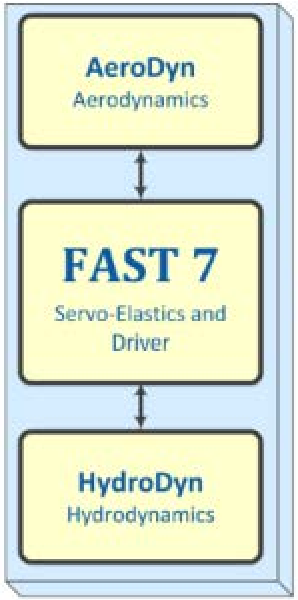
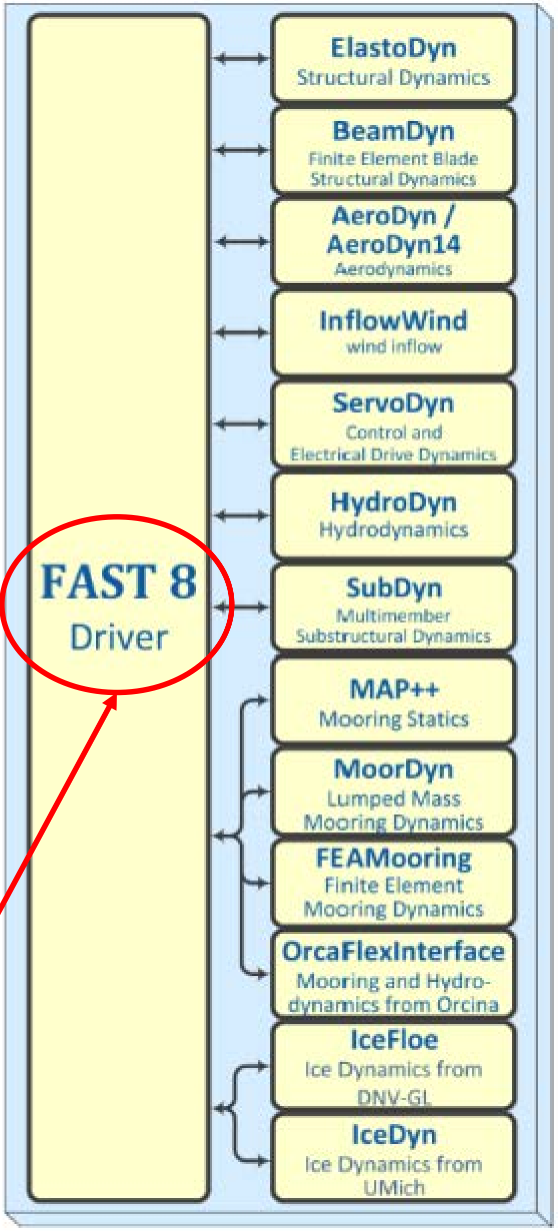
Introduction: The OpenFAST Multi-Physics Engineering Tool

- **OpenFAST** is DOE/NREL's premier open-source wind turbine multi-physics engineering tool
- **FAST** has undergone a major restructuring, w/ a new modularization framework (v8)
- Framework originally designed w/ intent of enabling full-system linearization, but functionality is being implemented in stages



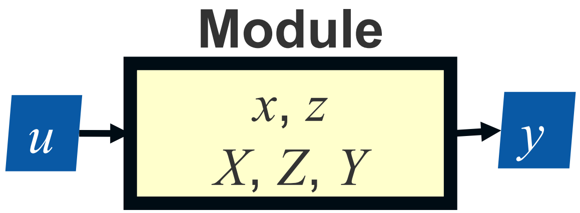


Now called **OpenFAST**



Introduction: Why Linearize?

- **OpenFAST** primary used for nonlinear time-domain standards-based load analysis (ultimate & fatigue)
- Linearization is about *understanding*:
  - Useful for eigenanalysis, controls design, stability analysis, gradients for optimization, & development of reduced-order models
- Prior focus:
  - Structuring source code to enable linearization
  - Developing general approach to linearizing mesh-mapping w/n module-to-module input-output coupling relationships, including rotations
  - Linearizing core (but not all) features of **InflowWind**, **ServoDyn**, **ElastoDyn**, & **AeroDyn** modules & their coupling
  - Verifying implementation
- This work – Linearizing **BeamDyn** & coupling
- Related work in parallel – Linearization for FOWT



$$\dot{x} = X(x,z,u,t)$$

$$0 = Z(x,z,u,t) \quad with \left|\frac{\partial Z}{\partial z}\right| \neq 0$$

$$y = Y(x,z,u,t)$$

$$u = u\big|_{op} + \Delta u \quad etc.$$

$$\Delta\dot{x} = A\Delta x + B\Delta u$$

$$\Delta y = C\Delta x + D\Delta u$$

$$with$$

$$A = \left[\frac{\partial X}{\partial x} - \frac{\partial X}{\partial z}\left[\frac{\partial Z}{\partial z}\right]^{-1}\frac{\partial Z}{\partial x}\right]\Bigg|_{op} \qquad etc.$$

Approach & Methods: Operating-Point Determination

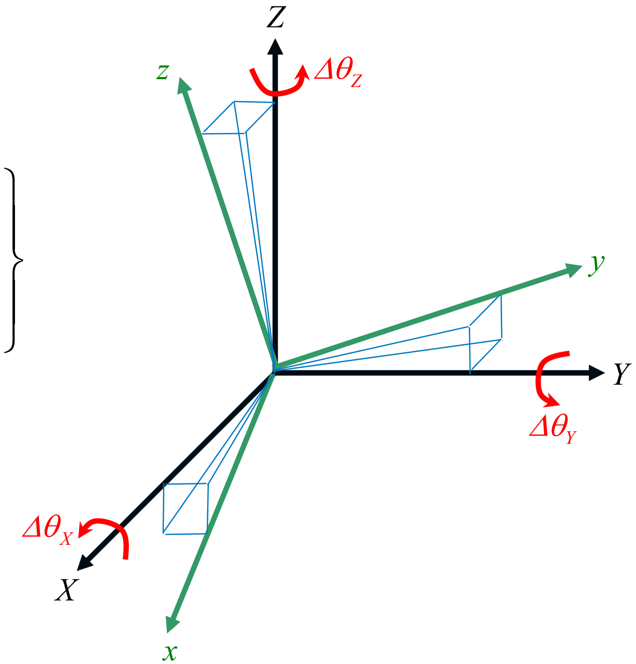
- A linear model of a nonlinear system is only valid in local vicinity of an operating point (OP)
- Current implementation allows OP to be set by given initial conditions (time zero) or a given times in nonlinear time-solution
- Note about rotations in 3D:
  - Rotations don't reside in a linear space
  - **FAST** framework stores module inputs/outputs for 3D rotations using 3×3 DCMs (Λ)
  - Linearized rotational parameters taken to be 3 small-angle rotations about global *X*, *Y*, & *Z* ($\Delta\vec{\theta}$)

$$u = u\big|_{op} + \Delta u \quad \textit{for most variables}$$

$$\Lambda = \Lambda\big|_{op}\, \Delta\Lambda \quad \textit{for rotations}$$

with

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \Lambda \begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix}$$



$$\Delta\Lambda = \begin{bmatrix} \frac{\Delta\theta_X^2\sqrt{1+\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2}+\Delta\theta_Y^2+\Delta\theta_Z^2}{\left(\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2\right)\sqrt{1+\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2}} & \frac{\Delta\theta_Z\left(\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2\right)+\Delta\theta_X\Delta\theta_Y\left(\sqrt{1+\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2}-1\right)}{\left(\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2\right)\sqrt{1+\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2}} & \frac{-\Delta\theta_Y\left(\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2\right)+\Delta\theta_X\Delta\theta_Z\left(\sqrt{1+\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2}-1\right)}{\left(\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2\right)\sqrt{1+\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2}} \\ \frac{-\Delta\theta_Z\left(\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2\right)+\Delta\theta_X\Delta\theta_Y\left(\sqrt{1+\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2}-1\right)}{\left(\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2\right)\sqrt{1+\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2}} & \frac{\Delta\theta_X^2+\Delta\theta_Y^2\sqrt{1+\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2}+\Delta\theta_Z^2}{\left(\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2\right)\sqrt{1+\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2}} & \frac{\Delta\theta_X\left(\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2\right)+\Delta\theta_Y\Delta\theta_Z\left(\sqrt{1+\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2}-1\right)}{\left(\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2\right)\sqrt{1+\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2}} \\ \frac{\Delta\theta_Y\left(\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2\right)+\Delta\theta_X\Delta\theta_Z\left(\sqrt{1+\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2}-1\right)}{\left(\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2\right)\sqrt{1+\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2}} & \frac{-\Delta\theta_X\left(\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2\right)+\Delta\theta_Y\Delta\theta_Z\left(\sqrt{1+\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2}-1\right)}{\left(\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2\right)\sqrt{1+\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2}} & \frac{\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2\sqrt{1+\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2}}{\left(\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2\right)\sqrt{1+\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2}} \end{bmatrix}$$

$$\Delta\Lambda \approx \begin{bmatrix} 1 & \Delta\theta_Z & -\Delta\theta_Y \\ -\Delta\theta_Z & 1 & \Delta\theta_X \\ \Delta\theta_Y & -\Delta\theta_X & 1 \end{bmatrix} \qquad \Delta\vec{\theta} = \begin{Bmatrix} \Delta\theta_X \\ \Delta\theta_Y \\ \Delta\theta_Z \end{Bmatrix}$$

Approach & Methods: Module Linearization

Module	Linear Features	States (x, z)	Inputs (u)	Outputs (y)	Jacobian Calc.
InflowWind (IfW)	• Uniform or steady wind	• None	• Positions • Wind parameter disturbances	• Undisturbed (inflow) wind @ input positions • User-selected wind-inflow outputs	• Analytical
ElastoDyn (ED)	• Structural dynamics of: ○ Blades ○ Drivetrain ○ Nacelle ○ Tower ○ Platform	• Structural degrees-of-freedom (DOFs) & their 1st time derivatives (continuous states)	• Applied loads along blades & tower • Applied loads on hub, nacelle, & platform • Blade-pitch-angle command • Nacelle-yaw moment • Generator torque	• Motions along blades & tower • Motions of hub, nacelle, & platform • Nacelle-yaw angle & rate • Generator speed • User-selected structural outputs (motions &/or loads)	• Numerical central-difference perturbation technique*
BeamDyn (BD)	• Structural dynamics of blades	• Structural degrees-of-freedom (DOFs) & their 1st time derivatives (continuous states)	• Motions of blade root • Applied loads along blade	• Blade-root reaction loads • Motions along blade • User-selected structural outputs (motions &/or loads)	• Numerical central-difference perturbation technique*
AeroDyn (AD)	• Aerodynamic stiffness & damping • BEM or frozen wake	• Inflow angle along blades (constraint states)	• Motions along blades & tower • Motions of hub • Undisturbed (inflow) wind along blades & tower	• Aerodynamic applied loads along blades & tower • User-selected aerodynamic outputs	• Numerical central-difference perturbation technique*

**Numerical central -difference perturbation technique (see paper for treatment of 3D rotations)*

$$\left.\frac{\partial X}{\partial x}\right|_{op} = \frac{X\left(\left.x\right|_{op} + \Delta x, \left.u\right|_{op}, \left.t\right|_{op}\right) - X\left(\left.x\right|_{op} - \Delta x, \left.u\right|_{op}, \left.t\right|_{op}\right)}{2\Delta x} \quad etc.$$

Approach & Methods: Rotation of States in BeamDyn

- Translational & rotational states in **BeamDyn** are defined globally in (nonrotating), but are oriented w/ root reference orientation
- For purposes of post-processing with **MBC3**, **BeamDyn** states can optionally be transformed to rotating frame during linearization

$$\Delta x^R = \underbrace{\begin{bmatrix} \Lambda^{Root}\big|_{op}\left[\Lambda^{RootR}\right]^T & 0 & 0 & 0 \\ 0 & \Lambda^{Root}\big|_{op}\left[\Lambda^{RootR}\right]^T & 0 & 0 \\ 0 & 0 & \Lambda^{Root}\big|_{op}\left[\Lambda^{RootR}\right]^T & 0 \\ 0 & 0 & 0 & \Lambda^{Root}\big|_{op}\left[\Lambda^{RootR}\right]^T \end{bmatrix}}_{T^R|_{op}} \Delta x$$

$$\Delta\dot{x} = A\Delta x + B\Delta u$$
$$\Delta y = C\Delta x + D\Delta u$$

$\Rightarrow$

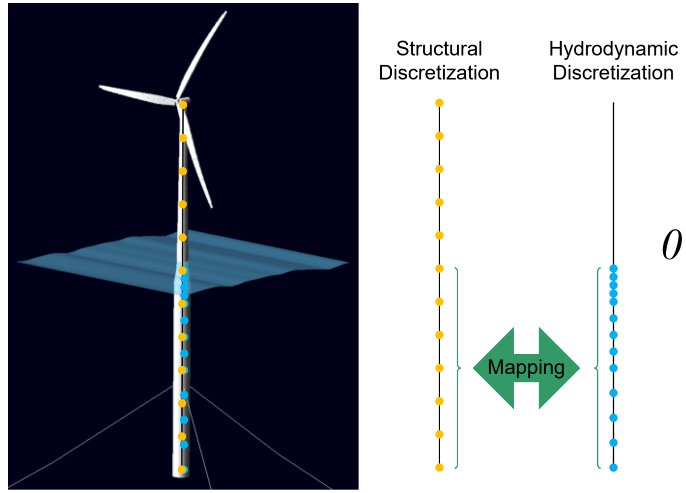
$$\Delta\dot{x}^R = A^R\Delta x^R + B^R\Delta u$$
$$\Delta y = C^R\Delta x^R + D^R\Delta u$$

with

$$A^R = T^R\big|_{op} A\left[T^R\big|_{op}\right]^T \qquad B^R = T^R\big|_{op} B$$
$$C^R = C\left[T^R\big|_{op}\right]^T \qquad D^R = D$$

Approach & Methods: Glue-Code Linearization

- Module inputs & outputs residing on spatial boundaries use a mesh, consisting of:
  - Nodes & elements (nodal connectivity)
  - Nodal reference locations (position & orientation)
  - One or more nodal fields, including motion, load, &/or scalar quantities
- Mesh-to-mesh mappings involve:
  - Mapping search – Nearest neighbors are found
  - Mapping transfer – Nodal fields are transferred
- Mapping transfers & other module-to-module input-output coupling relationships have been linearized analytically



$$0 = U\left(u, y\right)$$

$$0 = \left.\frac{\partial U}{\partial \tilde{u}}\right|_{op} \Delta u + \left.\frac{\partial U}{\partial y}\right|_{op} \Delta y$$

with

$$\Delta u = \begin{Bmatrix} \Delta u^{(IfW)} \\ \Delta u^{(SrvD)} \\ \Delta u^{(ED)} \\ \Delta u^{(\backslash BD)} \\ \Delta u^{(AD)} \end{Bmatrix}$$

$$\left.\frac{\partial U}{\partial \tilde{u}}\right|_{op} = \left.\begin{bmatrix} I & 0 & 0 & 0 & \frac{\partial U^{(IfW)}}{\partial \tilde{u}^{(AD)}} \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & I & \frac{\partial U^{(ED)}}{\partial \tilde{u}^{(BD)}} & \frac{\partial U^{(ED)}}{\partial \tilde{u}^{(AD)}} \\ 0 & 0 & 0 & \frac{\partial U^{(BD)}}{\partial \tilde{u}^{(BD)}} & \frac{\partial U^{(BD)}}{\partial \tilde{u}^{(AD)}} \\ 0 & 0 & 0 & 0 & \frac{\partial U^{(AD)}}{\partial \tilde{u}^{(AD)}} \end{bmatrix}\right|_{op}$$

etc.

Approach & Methods: Final Matrix Assembly

InflowWind (IfW)

$$\Delta y^{(IfW)} = D^{(IfW)} \Delta u^{(IfW)}$$

ServoDyn (SrvD)

$$\Delta y^{(SrvD)} = D^{(SrvD)} \Delta u^{(SrvD)}$$

AeroDyn (AD)

$$\Delta y^{(AD)} = D^{(AD)} \Delta u^{(AD)}$$

ElastoDyn (ED)

$$\Delta \dot{x}^{(ED)} = A^{(ED)} \Delta x^{(ED)} + B^{(ED)} \Delta u^{(ED)}$$
$$\Delta y^{(ED)} = C^{(ED)} \Delta x^{(ED)} + D^{(ED)} \Delta u^{(ED)}$$

BeamDyn (BD)

$$\Delta \dot{x}^{(BD)} = A^{(BD)} \Delta x^{(BD)} + B^{(BD)} \Delta u^{(BD)}$$
$$\Delta y^{(BD)} = C^{(BD)} \Delta x^{(BD)} + D^{(BD)} \Delta u^{(BD)}$$

Glue Code

$$0 = \left.\frac{\partial U}{\partial \tilde{u}}\right|_{op} \Delta u + \left.\frac{\partial U}{\partial y}\right|_{op} \Delta y$$

Full System

$$\Delta \dot{x} = A \Delta x + B \Delta u^{+} \qquad \Delta x = \begin{Bmatrix} \Delta x^{(ED)} \\ \Delta x^{(BD)} \end{Bmatrix} \qquad \Delta u^{+} = \begin{Bmatrix} \Delta u^{(IfW)} \\ \Delta u^{(SrvD)} \\ \Delta u^{(ED)} \\ \Delta u^{(\backslash BD)} \\ \Delta u^{(AD)} \end{Bmatrix}^{+} \qquad \Delta y = \begin{Bmatrix} \Delta y^{(IfW)} \\ \Delta y^{(SrvD)} \\ \Delta y^{(ED)} \\ \Delta y^{(\backslash BD)} \\ \Delta y^{(AD)} \end{Bmatrix}$$
$$\Delta y = C \Delta x + D \Delta u^{+}$$

with

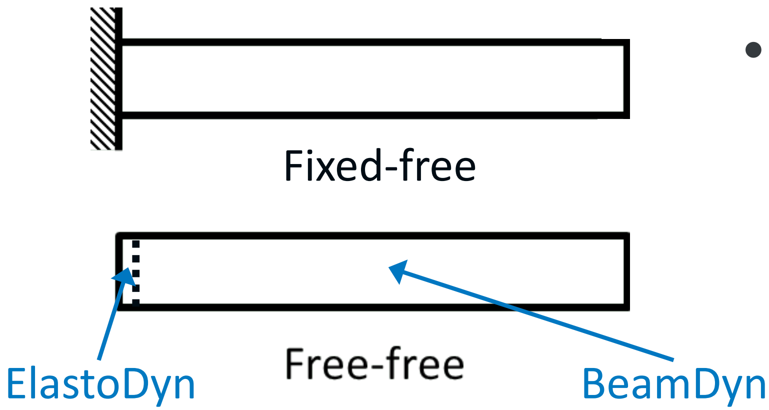
$$A = \begin{bmatrix} A^{(ED)} & 0 \\ 0 & A^{(BD)} \end{bmatrix} - \begin{bmatrix} 0 & 0 & B^{(ED)} & 0 & 0 \\ 0 & 0 & 0 & B^{(BD)} & 0 \end{bmatrix} \left[G|_{op} \right]^{-1} \left.\frac{\partial U}{\partial y}\right|_{op} \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ C^{(ED)} & 0 \\ 0 & C^{(BD)} \\ 0 & 0 \end{bmatrix}$$

etc.

$$G|_{op} = \left.\frac{\partial U}{\partial \tilde{u}}\right|_{op} + \left.\frac{\partial U}{\partial y}\right|_{op} \begin{bmatrix} D^{(IfW)} & 0 & 0 & 0 & 0 \\ 0 & D^{(SrvD)} & 0 & 0 & 0 \\ 0 & 0 & D^{(ED)} & 0 & 0 \\ 0 & 0 & 0 & D^{(BD)} & 0 \\ 0 & 0 & 0 & 0 & D^{(AD)} \end{bmatrix}$$

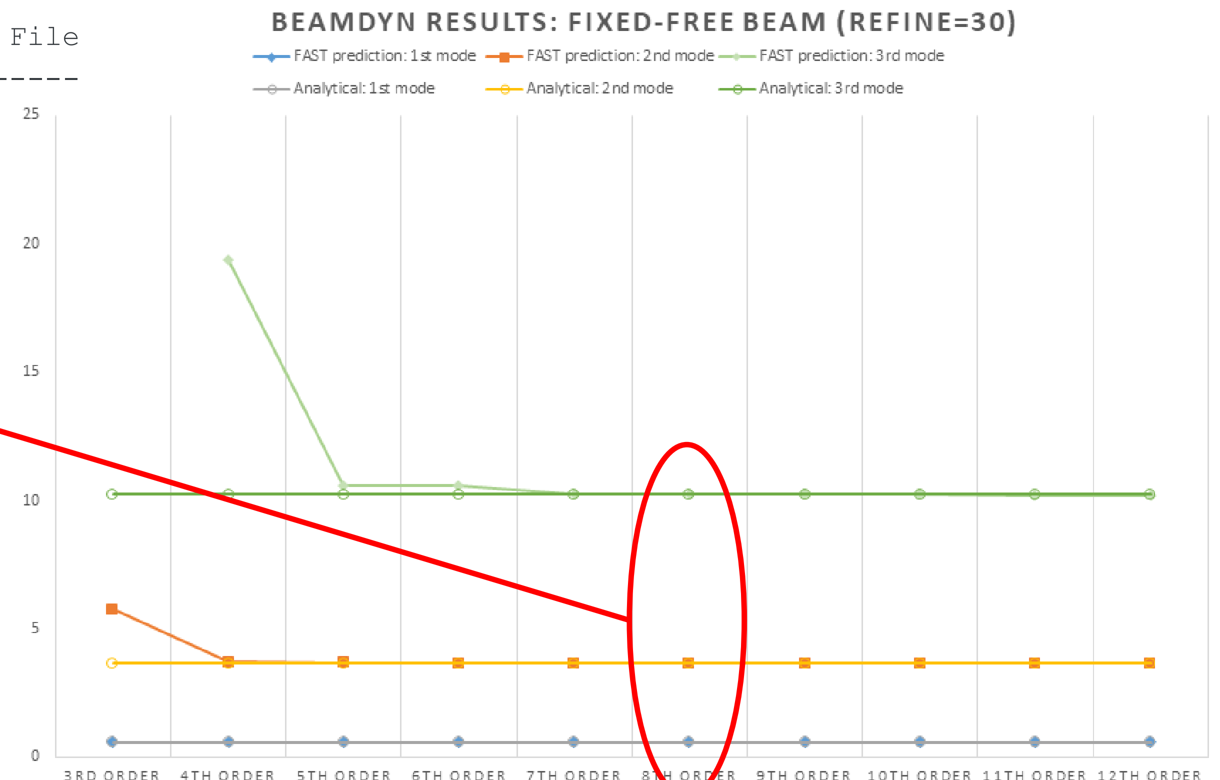
- D-matrices (included in G) impact all matrices of coupled system, highlighting important role of direct feedthrough
- While $A^{(ED)}$ contains mass, stiffness, & damping of **ElastoDyn** structural model only, full-system A contains mass, stiffness, & damping associated w/ full-system coupled aero-servo-elastics, including coupling to **BeamDyn** mass, stiffness, & damping & aerodynamic stiffness & damping

Results – Fixed-Free & Free-Free Beams



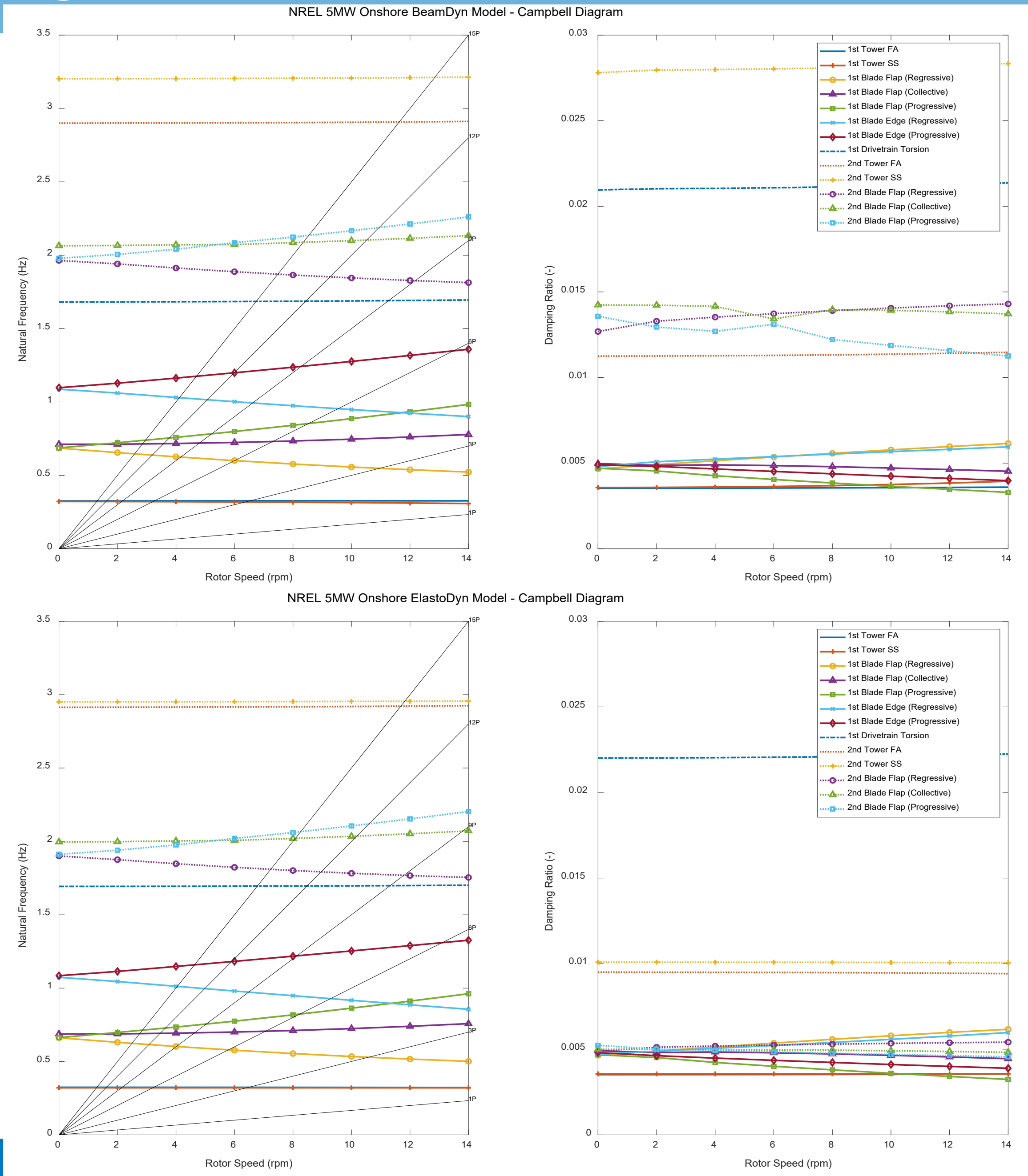
- Note: Strong sensitivity to output precision

Mode	Analytical	Lineariz'n	BD Summary File
Fixed-Free Beam (Hz):			
1	0.5842	0.5838	0.5838
2	0.5842	0.5938	0.5838
3	3.6607	3.6584	3.6583
4	3.6607	3.6584	3.6583
5	10.2512	10.2365	10.2364
6	10.2512	10.2365	10.2364
Free-Free Beam (Hz):			
1	3.7171	3.6579	
2	3.7171	3.6579	
3	10.2465	10.1808	
4	10.2465	10.1808	
5	20.0873	19.9459	
6	20.0873	19.9459	

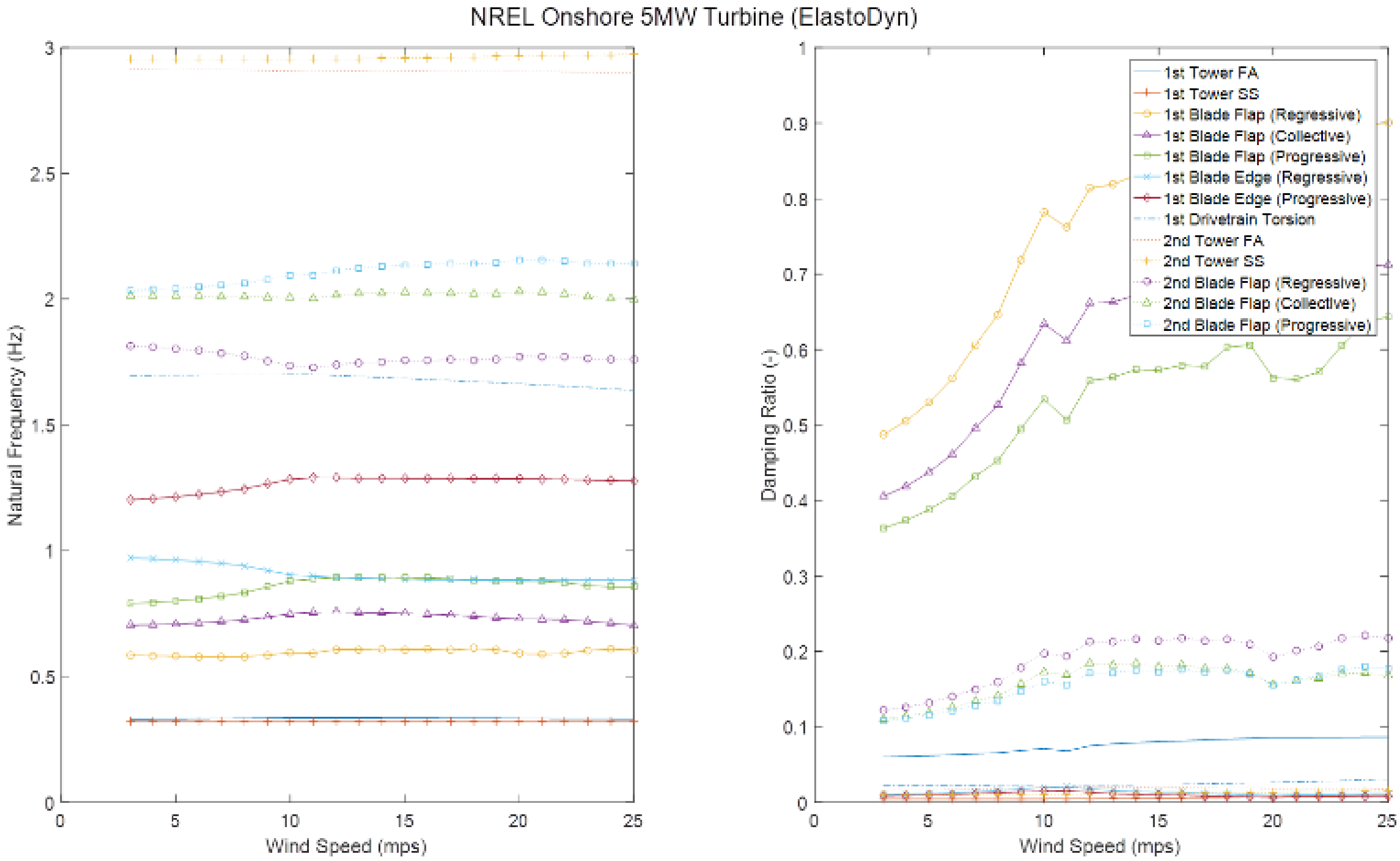


Results – Campbell Diagram of NREL 5-MW Turbine

- Modules enabled:
  - **ElastoDyn** or **ElastoDyn + BeamDyn**
  - **ServoDyn**
- Approach (for each rotor speed):
  1) Find periodic steady-state OP
  2) Linearize
  3) MBC
  4) Azimuth-average
  5) Eigenanalysis
  6) Extract natural frequencies & damping



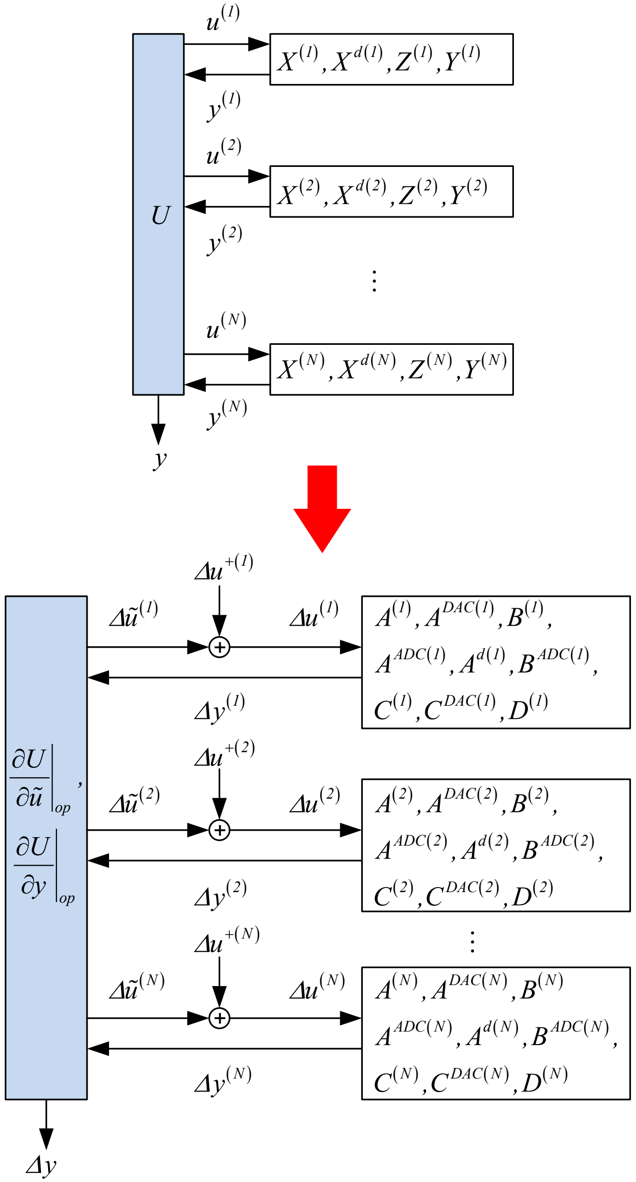
Results: Campbell Diagram of NREL 5-MW Turbine w/ Aero



- Modules enabled: **ElastoDyn** + **BeamDyn**, **ServoDyn**, **AeroDyn**
- Approach (for each wind speed): Define rotor speed & blade-pitch angle → Find periodic steady-state OP → Linearize → MBC → Azimuth-average → Eigenanalysis → Extract natural frequencies & damping ratios

Conclusions & Future Work

- Conclusions:
  - Linearization of underlying nonlinear wind-system equations advantageous to:
    - Understand system response
    - Exploit well-established methods/tools for analyzing linear systems
  - Linearization functionality has been expanded to aeroelastically tailored rotors w/n **OpenFAST**
- Future work:
  - Publish journal article on development & results
  - Improved OP through static-equilibrium, steady-state, or periodic steady-state determination
  - Eigenmode automation & visualization
  - Linearization functionality for:
    - Other important features (e.g. unsteady aerodynamics of **AeroDyn**)
    - Other offshore functionality (**SubDyn**, etc.)
    - New features as they are developed



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