Memorandum
Derivation and Description of the Soil-Pile-Interaction Models
IEA-Annex XXIII Subtask 2

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This memo describes relevant detail of the soil-pile-interaction models used for the monopile configuration within the code comparison exercise OC3. As decided by the participants only linearized soil-conditions based on the p-y-method are taken into account since no validated models for non-linear effects under dynamic loading are available.
The task here is to find a combination of both the monopile and the soil ensuring a proper participation of the soil structure interaction on the dynamic response. This can be achieved easily by application of very soft conditions. On the other hand certain limitations are given due to the resulting eigenfrequencies and tower top deflections of the dynamic response. Both, the design of the sub-soil part of the monopile as well as the selection of soil properties are based on realistic values and typical design procedures. Furthermore, the overall design can be considered as quite reasonable for a real site selection and design solution. The intention here is the verification of the available foundation modules of the simulation codes which requires an adequate participation of the soil-structure interaction models in the dynamic response rather than finding representative or even optimal soil conditions and monopile designs for an economic and reasonable solution.
Furthermore, auxiliary effects such as souring are neglected. The linearization of the p-y curves is performed for pre-loaded conditions where the response results from load case 5.3 of Phase 1. Three models are derived from the linearized p-y-curve approach. This memo refers to the description and derivation procedure of the soil conditions and models under consideration while the calculation results and model parameters are provided by additional documents.

1.Soil type and soil conditions

Numerous soil models in terms of p-y curves exists for different soil types and loading conditions. Here, only cyclic loading conditions are considered resulting in a range of relatively simple to complex non-linear behaviour which depends on the soil type.
Soil models for sand shows a relatively simple non-linear behaviour. Furthermore, such models are well described in certain standards and the derivation is not very complicated. Therefore, the API-sand model is taken into account. Based on the two soil properties effective unit weight and angel of internal friction together with the pile diameter the API standard [1] describes an easy procedure to derive the p-y curves over depth.

Here, layered soil conditions are considered in order to achieve both, realistic soil conditions as well as a certain participation of the resulting soil-structure interaction model in the dynamic response. The soil consists of three layers of sand with different properties in terms of internal friction angles. Figure 1 shows the soil profile and the properties of the single soil layers. The subsoil part of the monopile is simply an extension of the monopile above mudline. The densities of the single soil layer vary from medium dense to dense and the stiffness of the layers increase with depth. By this kind of layered soils with increasing stiffness of deeper layer the objective of a large participation of the soil-structure-interaction model in the dynamic response is fulfilled by the upper layer while the lower layer ensures a proper overall stiffness of the foundation.

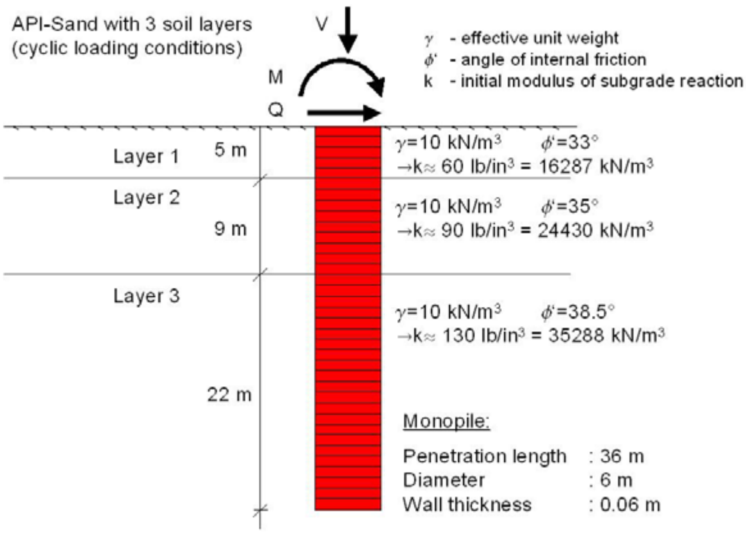

Figure 1: Soil profile

A selection of parameter and properties for soil layer 2 at a depth of 10 m below mudline is presented below.

Constant values (over depth of the layer):

Pile diameter D = 6.0 m
Internal friction angle ϕ = 35.0 deg.
Effective unit weight γ = 10.0 kN/ m^3
Loading condition factor A = 0.9

The coefficients C_1, C_2 and C_3 for calculation of the ultimate lateral bearing capacity over depth are given in figure 6.8.6-1 as a function of the internal friction angle.

C_1 = 2.97045
C_2 = 3.41918
C_3 = 53.79345

Furthermore the initial modulus of subgrade reaction k is given in figure 6.8.7-1 as a function of the internal friction angle.

Soil modulus parameter k = 24430 kN/m^3

This correlates to medium dense soil conditions i.e. a relative density of approx. 55 %.

Depth-dependent values:

Depth below ground surface = 10.0 m
Equivalent Depth = 9.55 m (required for layerd soils if $P_{us} \leq P_{ud}$)
Shallow depths P_{us} = 4669 kN/ m
Deep depths P_{ud} = 32276 kN/ m

From the given values the ultimate bearing capacity calculates as the lower value of Pus and Pud

P_u = 4669 kN/ m

All values are known to calculate the p-y curves at the desired heights. As an example the p-y curve as well as the relevant parameters for the level 10 m below mudline are given here:

Table 1 Discrete values of the p-y curve at 10 m below mudline

y [m]	0	0.0083	0.0167	0.025	0.0333	0.0417	0.05	0.0583	0.0667	0.075	>0.075
p [kN/m]	0	1817	3061	3709	4000	4121	4170	4189	4197	4200	4202

More details on the other depth stations and soil layers are provided within the file ***OC3-LPILE_Results.txt***.

2.Loading conditions and monopile design

The soil behaviour strongly depends on the acting loads. Increased loads results in decreased soil stiffness, if deflections exceed the (quasi-)linear portion of the p-y curves.
As agreed by the OC3 participants only linear soil-structure interaction models are taken into account for the project. The linearization procedure requires either representative loads or application of the initial stiffness of the p-y curves, i.e. tangent on the curve at y = 0 m. Since the approach with initial stiffness from the p-y curves overestimates the soil stiffness the approach with representative loads is used. These representative loads are the dynamic responses from phase run 5.3 in terms of the maximum overturning moment M and corresponding base shear Q on mudline level.
It should be mentioned, that these dynamic responses were calculated on a model with different modal properties compared to the model here with a flexible foundation. Therefore, an iterative approach with recalculation of the dynamic responses for the modified overall model would be a more consistent approach. Instead, the used loads from phase run 5.3 are simply scaled by a factor of 1.5. This scaling can also be understood as a kind of safety factor which is frequently applied on soil-structure interaction models when applying p-y curves. It should also be mentioned, that large uncertainties are connected to the p-y models for large diameter piles under cyclic loading as applied here. The consideration of this scaling factor as a kind of safety factor becomes even more reasonable, if one further takes into account that no verified p-y based soil-structure interaction models for dynamic loading conditions exists.

$M = 124385$ kNm
$Q = 3910$ kN

For the given soil and loading conditions the monopile is simply designed under consideration of the second zero crossing of the pile deflection shape using the pile properties given on mudline level, i.e. outer diameter D = 6 m and wall thickness t = 60 mm. Extending the pile below this second zero crossing will not result in an significant increase of the bearing capacity while pile lengths below this second zero crossing result in large pile head deflections. Calculation of the pile deflections requires a solution of the non-linear soil-pile interaction problem. Here, the geotechnical code LPILE 4.0 is used to solve the problem. Figure 2 shows the resulting deflection shape of a pile with 36 m penetration depth. The total deformations on mudline level for the given loads are:

$w = 0.022566$ m (deflection)
$\varphi = 0.002404$ rad (rotation)

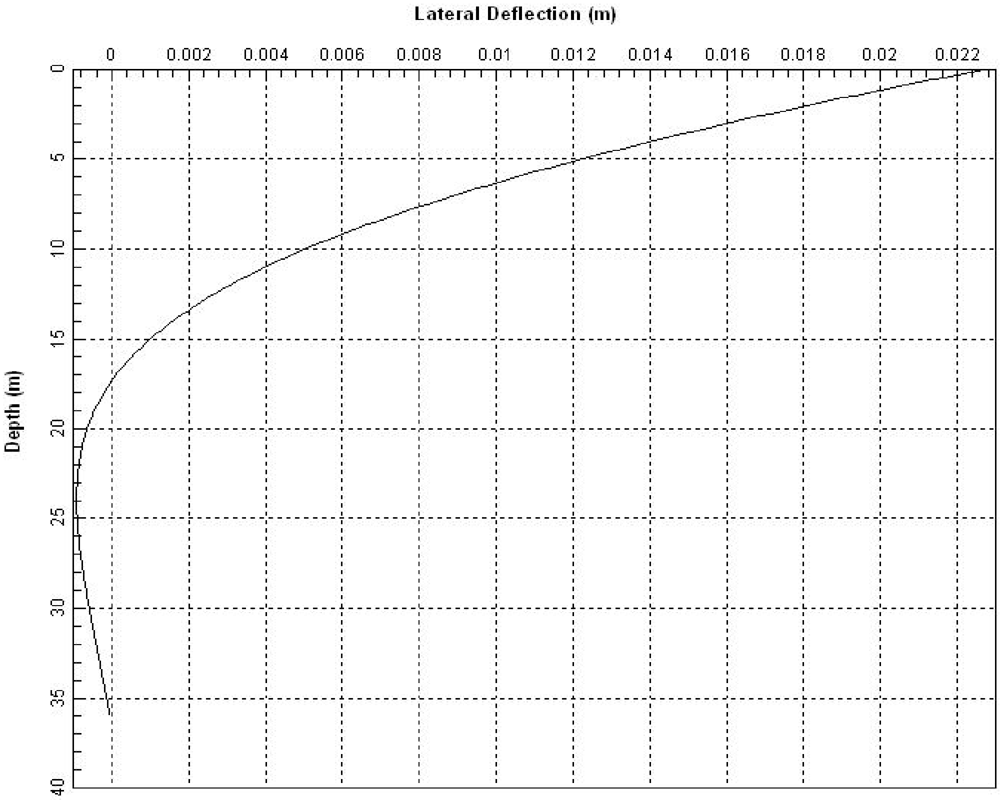

Figure 2 Deflection of the sub-soil portion of the pile

3.Linearized Monopile Foundation Models

Two different types of monopile foundation models are derived from the aforementioned loading conditions as shown in Figure 3. The first model is a modified form of the well-known apparent fixity length concept while the second model uses a set of coupled translational and rotational single springs resulting in a stiffness matrix coupling the degrees of freedom. The third model consists of the real sub-soil part of the monopile provided with lateral springs which are distributed over the length of the pile, linear and uncoupled. This model is denoted as the distributed springs (DS) model. All three models are linear i.e. based on a linearization of the non-linear soil-pile interactions.

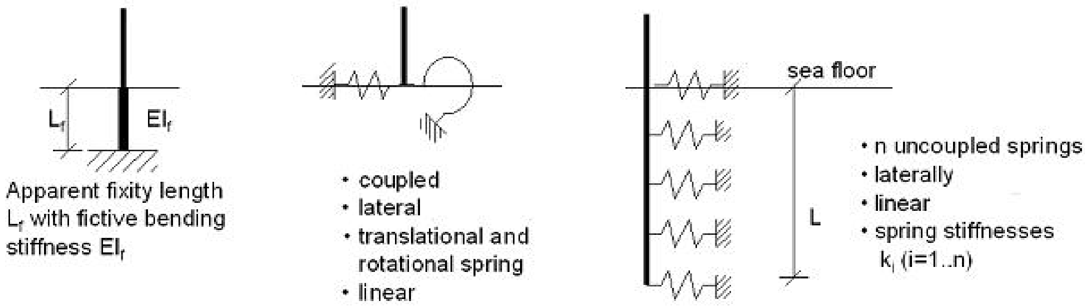

Figure 3 Monopile foundation models

3.1.Modified apparent fixity length approach

The well-known and widespread basic apparent fixity length model is a simple approach to substitute the more complex distributed spring model (DS) with a fictive fixed edge pile i.e. a cantilever beam. The structural and material properties of this fictive pile are the same as for the real pile on level of the sea floor i.e. on the interface between the real monopile and the fictive sub-soil part while the length of the fictive pile is determined iteratively in order to obtain a target 1^{st} eigenfrequency of the support structure. This target eigenfrequency may result from measurements or from calculations with more complex models. In general, good accuracy of the results in terms of modal properties and the dynamic response (only for the structure above mudline) is only obtained for stiff soil conditions compared to the pile stiffness. This condition is fulfilled for the pile and soil properties used here. However, the derivation of the apparent fixity length model follows an modified approach. Within the modified apparent fixity length concept the sub- soil part of the monopile is also replaced by a fictive pile that is fixed at the lower end. Here, (only) the geometrical properties of the fictive pile are adjusted in a straight-forward manner to meet the conditions at the interface point of the monopile and the fictive pile. These conditions are the deformations, i.e. deflection w and slope φ, as well as the loads, i.e. shear force and moment, as calculated for the complex model with LPILE4.0. To meet these conditions the following equations 1a and 1b for a clamped edge beam with a discrete shear force F and moment M on the free end as shown in Figure 4 must be fulfilled.

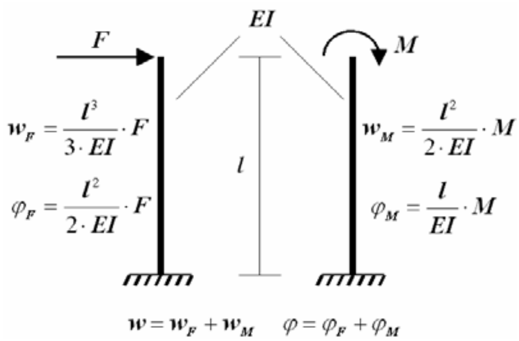

Figure 4 Clamped edge beam analogy

$$\frac{\boldsymbol{l}^3}{3 \cdot \boldsymbol{EI}} \cdot \boldsymbol{F} + \frac{\boldsymbol{l}^2}{2 \cdot \boldsymbol{EI}} \cdot \boldsymbol{M} = \boldsymbol{w} \quad \boldsymbol{Eq}.1\boldsymbol{a}$$

$$\frac{\boldsymbol{l}^2}{2 \cdot \boldsymbol{EI}} \cdot \boldsymbol{F} + \frac{\boldsymbol{l}}{\boldsymbol{EI}} \cdot \boldsymbol{M} = \varphi \quad \boldsymbol{Eq}.1\boldsymbol{b}$$

When dividing Eq.1a by Eq.1b and bringing the bringing the right side expression on the left side the following expression is obtained.

$$\frac{\boldsymbol{l}}{3} \cdot \frac{2 \cdot \boldsymbol{l} \cdot \boldsymbol{F} + 3 \cdot \boldsymbol{M}}{\boldsymbol{l} \cdot \boldsymbol{F} + 2 \cdot \boldsymbol{M}} - \frac{\boldsymbol{w}}{\varphi} = 0 \quad \boldsymbol{Eq}.2$$

Equation 2 is a 2^{nd} order polynominal of the fixity length l. Solution of the polynominal leads to the two expression given in equation 3.

$$L_{1,2}=\begin{bmatrix}\frac{1}{4\cdot\varphi\cdot F}\cdot\left(-3\cdot\varphi\cdot M+3\cdot w\cdot F+\left[9\cdot\varphi^2\cdot M^2+30\cdot\varphi\cdot w\cdot F\cdot M+9\cdot w^2\cdot F^2\right]^{0.5}\right)\\ \frac{1}{4\cdot\varphi\cdot F}\cdot\left(-3\cdot\varphi\cdot M+3\cdot w\cdot F-\left[9\cdot\varphi^2\cdot M^2+30\cdot\varphi\cdot w\cdot F\cdot M+9\cdot w^2\cdot F^2\right]^{0.5}\right)\end{bmatrix}\quad \textit{Eq.}3$$

In general either l_1 or l_2 is positive while the other value is negative. However, at least one positive value should be found for l and the fictive bending stiffness can then be evaluated by equation *4a* or *4b* derived from equations *1a* and *1b*.

$$EI=\frac{F\cdot l^3}{3\cdot w}+\frac{M\cdot l^2}{2\cdot w}\quad \textit{Eq.}4a$$

$$EI=\frac{F\cdot l^2}{2\cdot\varphi}+\frac{M\cdot l}{\varphi}\quad \textit{Eq.}4b$$

Finally, the mass distribution of the fictive pile should result in approximately the same total mass as the real sub-soil pile.

3.2.Coupled single springs

The model with coupled single springs is a stiffness matrix representation for the soil-structure interaction model. The stiffness matrix has the following form:

$$\begin{bmatrix}k_{u,F} & k_{u,M}\\ k_{\varphi,F} & k_{\varphi,m}\end{bmatrix}\begin{bmatrix}w\\ \varphi\end{bmatrix}=\begin{bmatrix}F\\ M\end{bmatrix}\quad \textit{Eq.}\ 5$$

The physical meaning of the matrix representation is that due to the lateral load F not only a lateral deflection u_F, but also a rotation φ_F occurs. A moment M on the other hand will not only cause a rotation φ_M, but also a deflection u_M. Here, the analogy to the aforementioned cantilever beam is obvious. A common approach for the derivation of the linearized stiffness matrix coefficients that takes the non-linear behaviour of the soil-structure interactions in an averaging manner into account is described by [2]. In order to achieve a more consistent model with respect to the aforementioned modelling approaches with distributed springs and a cantilever beam an alternative approach is taken here. In principle, the model is described in the previous section. By writing eq. 1a and 1b in matrix form the following expression is achieved

$$\begin{bmatrix}\frac{l^3}{3\cdot EI} & \frac{l^2}{2\cdot EI}\\ \frac{l^2}{2\cdot EI} & \frac{l}{EI}\end{bmatrix}\begin{bmatrix}F\\ M\end{bmatrix}=\begin{bmatrix}w\\ \varphi\end{bmatrix}\quad \textit{Eq.}\ 6$$

The left hand side matrix is the flexibility matrix where the values have already been derived in the previous section. Inverting this flexibility matrix provide the numerical values of the stiffness matrix.

$$\begin{bmatrix}\frac{l^3}{3\cdot EI} & \frac{l^2}{2\cdot EI}\\ \frac{l^2}{2\cdot EI} & \frac{l}{EI}\end{bmatrix}^{-1}\begin{bmatrix}w\\ \varphi\end{bmatrix}=\begin{bmatrix}F\\ M\end{bmatrix}\quad \textit{Eq.}\ 7$$

The stiffness matrix in eq. 7 corresponds to the formulation of the stiffness matrix for a Bernoulli beam as given in eq. 8:

$$\begin{bmatrix}\frac{12\cdot EI}{l^3} & -\frac{6\cdot EI}{l^2}\\ -\frac{6\cdot EI}{l^2} & \frac{4\cdot EI}{l}\end{bmatrix}\begin{bmatrix}w\\ \varphi\end{bmatrix}=\begin{bmatrix}F\\ M\end{bmatrix}\quad \textit{Eq.}\ 8$$

Within this approach no participation of mass is considered for the subsoil part of the pile. Mass properties can be accounted for by discrete mass and inertia elements on sea floor level. However, the contribution of the subsoil part of the pile to the modal properties (for the modes of interest) is predominantly given by the stiffness properties.

3.3.Distributed springs model

Within this approach the linearized soil-pile interactions are approximated by single, linear, translational springs which are distributed along the length of the sub-soil part of the pile i.e. the linearized model presented in Figure 5. Evaluation of each spring stiffness requires the calculated values of the soil resistance and the deflections. Good accuracy can be expected when applying springs in approx. 1 m intervals along the pile. However, the pile is discretized much more detailed within LPILE4.0. Both, soil resistance per unit length p [kN/m] as well as deflections y [m] are calculated at each increment node. Division of the soil resistance per unit length p by the deflections y results in a distribution of the embedded spring stiffness[1] k [kN/m^2]. This embedded spring stiffness must now be transformed into discrete springs on the desired locations of the pile as illustrated by Figure 5.

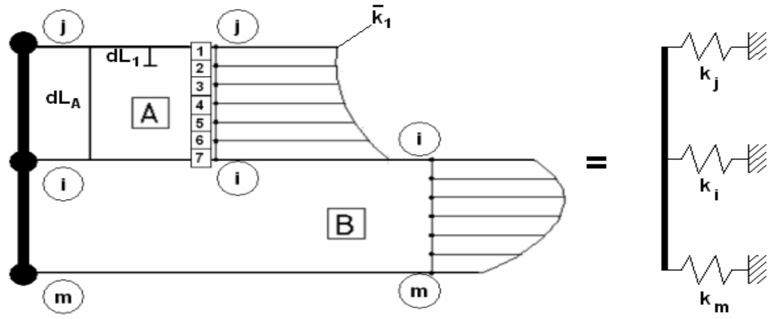

Figure 5 Approximation of discrete soil springs from spring stiffness distribution along the embedded monopile

4. New output sensors

Monopiles transfers the lateral loads to the surrounding soil over depth as shown in figure 6.
Therefore an increasing bending moment occurs in the upper part of the pile below mudline. The figure also shows that the maximum bending moment occurs at approx. 6.4 m below mudline which corresponds to 18 % of the pile penetration length. Additional sensors for the bending moment and pile deflection should therefore be located in that region.

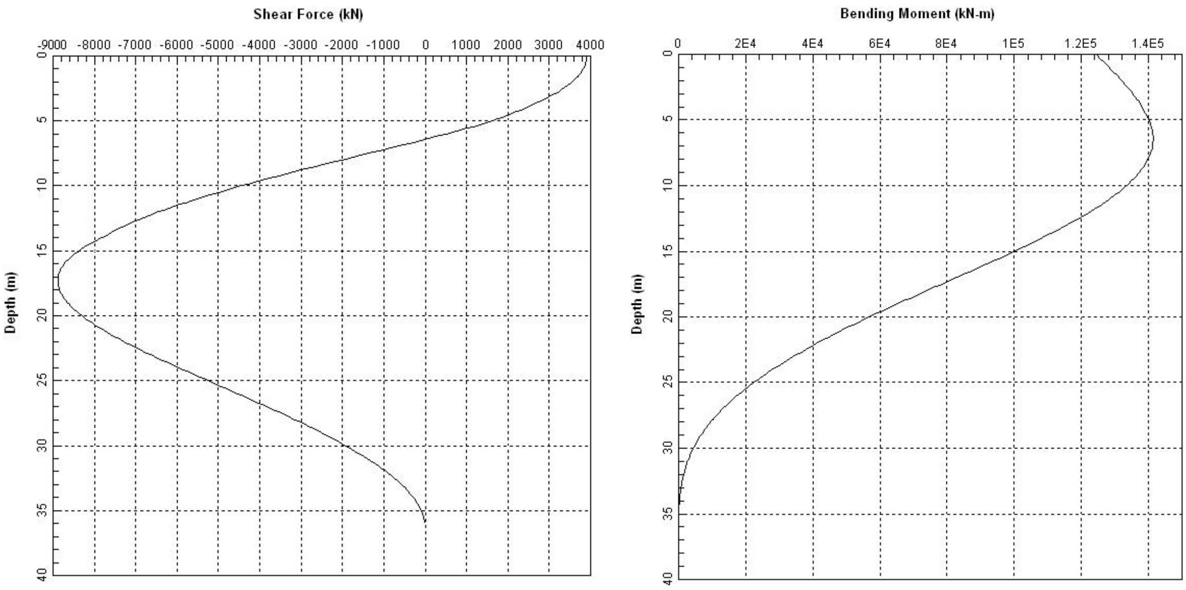

Figure 6 Shear force (left) and bending moment (right) over depth of the subsoil part of the monopile

[1] This correlates to the so-called secant stiffness. An alternative approach is the application of the so-called tangent stiffness. Due to the relatively small deflections which are mainly in the linear portion of the p-y curves both approaches results in approximately the same overall stiffness.

5.Simulation of the model with the flexible foundation

In this section results of a simulation are presented using conditions from load case 5.3 of phase 1 to get an impression of the dynamic behaviour of the overall structure with the flexible foundation. The calculations are performed using the SWE version of the Flex5 code and the modified apparent fixity length (AFL) approach to model the soil-structure interactions.

A selection of simulation results for load case 5.3 is given in table 2 for the flexible foundation model and in table 3 for the original rigid foundation. By comparison of both tables it becomes obvious, that the responses increases for the flexible foundation model.

Table 2 shows a maximum tower top deflection of 0.6 m for the flexible foundation model which is regarded within an acceptable range.
As mentioned in the introduction the soil-pile configurations must be chosen under consideration of the resulting eigenfrequencies for the support structure. Softer soil-pile configurations results in larger participation of the foundation (soil-pile interaction) on the dynamic response as well as in lower eigenfrequencies. Here, the range of possible 1st eigenfrequencies for the support structure is simply determined on basis of the Campbell diagram. Taking into account a range of rotor speeds from 6.9 to 12.1 rpm a frequency range from 0.222 Hz to 0.314 Hz (10% frequency offset from the exciting 1p and 3p frequencies) for a soft-stiff[2] design is possible.

Table 2 Simulation results for load case 5.3 using the flexible foundation model

Parameter	Tower Top Fore-Aft Deflection	Mudline Fore-Aft Shear Force	Mudline Fore-Aft Bending Moment
	TTDspFA	TwrBsFxt	TwrBsMyt
Units	(m)	(kN)	(kN·m)
Minimum	0.0526	-2198.5	-10299.7
Mean	0.3523	330.1	38407.0
Maximum	0.6036	2883.4	76215.5
StandDev	0.0855	760. 3	13475.6
DEL, m=5		1936.1	33432.8
DEL, m=10		2822.2	48021.3

Table 3 Simulation results for load case 5.3 using the rigid foundation model

Parameter	Tower Top Fore-Aft Deflection	Mudline Fore-Aft Shear Force	Mudline Fore-Aft Bending Moment
	TTDspFA	TwrBsFxt	TwrBsMyt
Units	(m)	(kN)	(kN·m)
Minimum	0.1737	-2192.9	-880.9
Mean	0.2774	330.0	38188.2
Maximum	0.4014	2815.7	82923.6
StandDev	0.0351	738.0	10692.9
DEL, m=5		1907.3	30078.0
DEL, m=10		2786.6	45311.0

The resulting eigenfrequencies of the support structure for the flexible foundation model are presented in table 4.

Table 4 Support structure eigenfrequencies for the flexible foundation model

Full System Eigenmode	Natural Frequency (Hz)
1st Tower Fore-Aft	0.2456
1st Tower Side-to-Side	0.2476
2nd Tower Fore-Aft	1.5327
2nd Tower Side-to-Side	1.5459

[2] Due to several reasons both, soft-soft and stiff-stiff designs seems to be inadequate for the design of an appropriate soil-structure interaction model for our purposes.

6.Summary

The soil consists of three layers of sand with different properties in terms of internal friction angles. Table 4 gives the soil profile and summarizes the properties of the single soil layers. Layer top and layer bottom are measured from the mudline.

Table 4 Properties of the single soil layer

	Layer 1	Layer 2	Layer 3
Soil type	API sand	API sand	API sand
Loading type	cyclic	cyclic	cyclic
Layer top [m]	0	5	14
Layer bottom [m]	5	14	∞
Effective unit weight γ [kN/m^3]	10	10	10
Internal friction angle ϕ [°]	33	35	38.5
p-y modulus k [kN/m^3]	16287	24430	35288
Coefficient C_1 [-]	2.49133	2.97045	4.04577
Coefficient C_2 [-]	3.09732	3.41918	4.06556
Coefficient C_3 [-]	41.72551	53.79345	85.05375

Pile properties (sub-soil portion of the pile)

Pile diameter D: 6 m (constant)
Wall thickness t: 60 mm (constant)
Penetration depth l: 36 m (pile length below mudline)
Denstiy: 8500 kg/m^3
Modulus of elasticity: $2.1 \bullet 10^{11}$ N/m^2

The geometrical and material properties of the sub-soil portion of the pile are the same as for the monopile above mudline. The only additional property is the pile penetration depth of 36 m.

Foundation model with distributed, linear springs

Single, linear, lateral springs are applied in 1 m intervals to the sub-soil part of the pile, resulting in 37 single springs in total. The single spring stiffnesses are provided in the Excel-File *OC3-Soil-Pile_InteractionModels.xls* on the spreadsheet *distributed springs*.

Apparent fixity length foundation model

This foundation model is a more simple representation of the soil-structure interactions and intended for application within codes that do not support the aforementioned, more complex model with distributed springs. Here, the soil-structure interaction model is represented by a fictive beam that is fixed at the lower end, here at 17.5022 m below mudline. The properties of this fictive beam are summarized below:

Pile diameter D: 6.2132 m (constant)
Wall thickness t: 59.868 mm (constant)
Length l: 17.50 m (pile length below mudline)
Density: 8500 kg/m^3
Modulus of elasticity: $2.1 \bullet 10^{11}$ N/m^2

From the given values the fictive bending stiffness Ei_f and the fictive mass per unit length μ_f can be derived:

$Ei_f = 1.15 \bullet 10^{12}$ N/m^2
$\mu_f = 9837.2$ kg/m

Single coupled springs (stiffness matrix) on sea floor level

The foundation stiffness matrix has the following form:

$$\begin{bmatrix} \boldsymbol{k}_{u,F} & \boldsymbol{k}_{u,M} \\ \boldsymbol{k}_{\varphi,F} & \boldsymbol{k}_{\varphi,m} \end{bmatrix} \begin{bmatrix} \boldsymbol{w} \\ \varphi \end{bmatrix} = \begin{bmatrix} \boldsymbol{F} \\ \boldsymbol{M} \end{bmatrix}$$

The coefficients of the stiffness matrix are derived as :

$k_{u,F}$	=	2.58E+09	N/m
$k_{u,M}$=($k_{\varphi,F}$)	=	-2.26E+10	N/rad
$k_{\varphi,F}$=($k_{u,M}$)	=	-2.26E+10	Nm/m
$k_{\varphi,M}$	=	2.64E+11	Nm/rad

Additional documents

OC3-Soil-Pile_InteractionModels.xls :
Excel-Spreadsheet containing model data for the introduced foundation models and some validation data in terms of statics and dynamics.

OC3-Soil-Pile-Interaction Models_ReadMe.pdf :
Short note on the soil properties, pile properties as well as the soil-structure interaction models.

OC3-LPILE_Results.txt :
LPILE output file, containing data about the pile, soil, p-y-curves for some depths and some more calculated values.

References

[1] American Petroleum Institute; Recommended Practice for Planning, Designing and Constructing Fixed Offshore Platforms Working Stress Design
API RECOMMENDED PRACTICE 2A-WSD (RP 2A-WSD) TWENTY-FIRST EDITION, DECEMBER 2000

[2] Reese, L. et al.; LpilePlus 4.0 for Windows, Technical Manual “ A Program for the Analysis of Piles and Drilled Shafts Under Lateral Loads”; Ensoft, Inc., Engineering Software; October 2000